ON LOWER CONFIDENCE BOUNDS FOR PCS
IN TRUNCATED LOCATION PARAMETER MODELS *

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1

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ON LOWER CONFIDENCE BOUNDS FOR PCS IN TRUNCATED LOCATION PARAMETER MODELS

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ABSTRACT

We are concerned with deriving lower confidence bounds for the probability of a correct selection in truncated location-parameter models. Two cases are considered according to whether the scale parameter is known or unknown. For each case, a lower confidence bound for the difference between the best and the second best is obtained. These lower confidence bounds are used to construct lower confidence bounds for the probability of a correct selection. The results are then applied to the problem of selecting the best exponential population having the largest truncated location-parameter. Useful tables are provided for implementing the proposed methods.

Key Words and Phrases: correct selection, probability of a correct selection, indifference zone, lower confidence bound, best population, truncated-location model, two-parameter exponential distribution.

1. Introduction

Let $X_{ij}, j = 1, \ldots, n$, be a sample of size n from a population π_i, where $\pi_1, \ldots, \pi_k$ are independently distributed with absolutely continuous cumulative distribution $G\left(\frac{x-\theta_i}{\beta}\right)$, $1 \leq i \leq k$, respectively, where $\beta > 0, -\infty < \theta_i < \infty, i = 1, \ldots, k$ and $G(x) = 0$ if $x \leq 0$. Let $\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_k)$ and let $\theta_{(1)} \leq \ldots \leq \theta_{(k)}$ denote the ordered values of $\theta_1, \ldots, \theta_k$. It is assumed that the exact pairing between the ordered and the unordered parameter is unknown. The population associated with $\theta_{(k)}$ is referred to as the best population. Assume that the experimenter is interested in the selection of the best population. For this purpose, let $X_i = \min(X_{i1}, \ldots, X_{in})$. We denote the cumulative distribution and the density function of X_i by $F\left(\frac{x-\theta_i}{\beta}\right)$ and $f\left(\frac{x-\theta_i}{\beta}\right)$, respectively. In many situations, X_i can be a sufficient statistic for the parameter θ_i. The natural selection rule is to select the population yielding the largest X_i as the best population. Thus, a question which arises naturally is: Is the selected population actually the best? Or, more precisely, what kind of confidence statement can be made regarding this selection?

Let CS (correct selection) denote the event that the best population is selected. Thus, the probability of a correct selection (PCS) at $\underset{\sim}{\theta}$ by applying the natural selection rule is:

$$\text{PCS } (\underset{\sim}{\theta}) = \int_{x=0}^{\infty} \overset{k-1}{\underset{j=1}{\pi}} F\left(x + \frac{\theta_{(k)} - \theta_{(j)}}{\beta}\right) dF(x). \tag{1.1}$$

In general, to guarantee the probability of a correct selection, one needs to specify a positive number δ^* such that $\theta_{(k)} - \theta_{(k-1)} \geq \delta^*$, see Bechhofer (1954). Clearly, this indifference zone approach is formulated on the basis of designing an experiment. However, in a real situation, it may be hard to assign the value of δ^* such that $\theta_{(k)} - \theta_{(k-1)} \geq \delta^*$, since the parameter values $\theta_1, \ldots, \theta_k$ are unknown. So that if the above assumption is not satisfied,

the probability of a correct selection cannot be guaranteed to be at least equal to the prespecified level. Parnes and Srinivasan (1986) have pointed out certain inconsistencies in the indifference zone formulation of certain selection problems. Also, see Fabian (1962) and Hsu (1981) for some possible ways to be out of this impasse.

Recently, retrospective analyses regarding the PCS have been studied by several authors. Olkin, Sobel and Tong (1976, 1982) have presented estimators of the PCS. Faltin and McCulloch (1983) have studied the small-sample properties of the Olkin-Sobel-Tong estimator of the PCS for the case when $k = 2$. Bofinger (1985) has discussed the nonexistence of consistent estimators of the PCS. Gutmann and Maymin (1987) have presented a procedure to test whether the selected population is the best. Anderson, Bishop and Dudewicz (1977) have given a lower confidence bound on the PCS in normal distribution models. Kim (1986) has presented a lower confidence bound on the PCS for the location-parameter model for the case where the underlying density function has the monotone likelihood ratio property and studied its application to normal model case. Gupta and Liang (1987) have derived a lower confidence bound for the PCS for the general location-parameter model and applied the result to normal populations.

In this paper, we are concerned with deriving lower confidence bounds for the probability of a correct selection for truncated location-parameter models. Two cases are considered according to whether the scale parameter β is known or unknown. For each of these, a lower confidence bound for the difference $(\theta_{(k)} - \theta_{(k-1)})/\beta$ is obtained and used to construct a lower confidence bound for the probability of a correct selection. The results are then applied to the problem of selecting the best two-parameter exponential

population having the largest truncated location parameter. Useful tables are provided for implementing the proposed procedures.

2. Lower Confidence Bound on PCS

In (1.1), replacing $\theta_{(k)} - \theta_{(i)}$ by $\theta_{(k)} - \theta_{(k-1)}$ for each $i = 1, \ldots, k-2$, we have

$$\text{PCS}\ (\underset{\sim}{\theta}) \geq \int_{x=0}^{\infty} \left[F\left(x + \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \right]^{k-1} dF(x). \tag{2.1}$$

Thus, if a lower confidence bound, say $\hat{\Delta}$, for $\frac{\theta_{(k)}-\theta_{(k-1)}}{\beta}$ can be obtained, then $\hat{P}_L = \int_{x=0}^{\infty}[F(x+\hat{\Delta})]^{k-1}dF(x)$ is a lower confidence bound for the PCS $(\underset{\sim}{\theta})$. In the following, lower confidence bounds for the $\frac{\theta_{(k)}-\theta_{(k-1)}}{\beta}$ are derived for the two cases where the scale parameter β is either known or unknown.

Let $X_{[1]} \leq \ldots \leq X_{[k]}$ denote the ordered statistics of $X_1, \ldots, X_k$, and let $X_{(i)}$ denote the random observation associated with $\theta_{(i)}, i = 1, \ldots, k$. For each fixed $\underset{\sim}{\theta}$, let $\underset{\sim}{\theta}^0 = (\theta_1^0, \ldots, \theta_k^0)$ where $\theta_i^0 = -\infty$ for $i = 1, \ldots, k-2$, and $\theta_i^0 = \theta_{(i)}$, for $i = k-1, k$. Then we obtain the following lemma which is analogous to Lemma 1 of Kim (1986).

Lemma 2.1. Let $f(y)$ be the density function of $F(y)$. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, for each fixed $c > 0$,

a) $P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\}$ is nonincreasing in $\theta_{(1)}$, and therefore,

b) $P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\} \leq P_{\underset{\sim}{\theta}^0}\{|X_{(k)} - X_{(k-1)}| > c\}$ for all $\underset{\sim}{\theta}$.

Proof: Without loss of generality, we may assume that $\theta_1 \leq \ldots \leq \theta_k$. Then, for each fixed

$c > 0$, straightforward computation leads to

$$\begin{aligned}
&P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\} \\
&= \sum_{i=1}^{k} P_{\underset{\sim}{\theta}}\{X_j \leq X_i - c \text{ for all } j \neq i\} \\
&= \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \prod_{j=2}^{k} F\left(y + \frac{\theta_1-\theta_j-c}{\beta}\right) dF(y) + \sum_{i=2}^{k-1} \int_{(\theta_k-\theta_i+c)/\beta}^{\infty} \prod_{\substack{j=1\\ j\neq i}}^{k} F\left(y+\frac{\theta_i-\theta_j-c}{\beta}\right) dF(y) \\
&\quad + \int_{\max((\theta_{k-1}-\theta_k+c)/\beta,0)}^{\infty} \prod_{j=1}^{k-1} F\left(y+\frac{\theta_k-\theta_j-c}{\beta}\right) dF(y).
\end{aligned}$$

Then,

$$\begin{aligned}
&\frac{\partial}{\partial\theta_1} P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\} \\
&= \sum_{j=2}^{k-1} \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1}\left[\prod_{\substack{m=2\\ m\neq j}}^{k} F\left(y+\frac{\theta_1-\theta_m-c}{\beta}\right)\right]\left[f\left(y+\frac{\theta_1-\theta_j-c}{\beta}\right)f(y) - \right. \\
&\qquad \left. f\left(y-\frac{c}{\beta}\right) f\left(y+\frac{\theta_1-\theta_j}{\beta}\right)\right] dy \\
&\quad + \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1}\left[\prod_{m=2}^{k-1} F\left(y+\frac{\theta_1-\theta_m-c}{\beta}\right)\right] f\left(y+\frac{\theta_1-\theta_k-c}{\beta}\right) f(y)dy \\
&\quad - \int_{\max((\theta_{k-1}-\theta_k+c)/\beta,0)}^{\infty} \beta^{-1}\left[\prod_{m=2}^{k-1} F\left(y+\frac{\theta_k-\theta_m-c}{\beta}\right)\right] f\left(y+\frac{\theta_k-\theta_1-c}{\beta}\right) f(y)dy \\
&\quad \equiv I_1 + I_2 - I_3 \text{ (say).}
\end{aligned}$$

Note that under the assumption, $f(y+\frac{\theta_1-\theta_j-c}{\beta})f(y) \leq f(y-\frac{c}{\beta})f(y+\frac{\theta_1-\theta_j}{\beta})$ for all $y \geq \frac{\theta_k-\theta_1+c}{\beta}$ since $y+\frac{\theta_1-\theta_j-c}{\beta} \leq y-\frac{c}{\beta}, y+\frac{\theta_1-\theta_j}{\beta} \leq y$. Thus, $I_1 \leq 0$. For the difference $I_2 - I_3$, we consider the following two cases.

Case 1. As $\theta_{k-1} - \theta_k + c > 0$, after changing variables,

$$\begin{aligned}
& I_2 - I_3 \\
&= \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y + \frac{\theta_1 - \theta_k - c}{\beta}\right) f(y) dy \\
&- \int_{(\theta_{k-1}-\theta_1+c)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y - \frac{c}{\beta}\right) f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right) dy \\
&= \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] \left[f\left(y + \frac{\theta_1 - \theta_k - c}{\beta}\right) f(y) - f\left(y - \frac{c}{\beta}\right) \right. \\
&\qquad \left. f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right)\right] dy \\
&- \int_{(\theta_{k-1}-\theta_1+c)/\beta}^{(\theta_k-\theta_1+c)/\beta} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y - \frac{c}{\beta}\right) f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right) dy \\
&\leq 0
\end{aligned} \tag{2.2}$$

since at the right-hand side of (2.2), the first term is nonpositive, see the preceding arguments, and the second term is nonnegative.

Case 2. As $\theta_{k-1} - \theta_k + c \leq 0$, changing variables and following straight computation, we have

$$\begin{aligned}
& I_2 - I_3 \\
&= \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y + \frac{\theta_1 - \theta_k + c}{\beta}\right) f(y) dy \\
&- \int_{(\theta_k-\theta_1)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y - \frac{c}{\beta}\right) f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right) dy \\
&= \int_{(\theta_k-\theta_1+c)/\beta}^{\infty} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] \left[f\left(y + \frac{\theta_1 - \theta_k - c}{\beta}\right) f(y) - f\left(y - \frac{c}{\beta}\right) \right. \\
&\qquad \left. f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right)\right] dy \\
&- \int_{(\theta_k-\theta_1)/\beta}^{(\theta_k-\theta_1+c)/\beta} \beta^{-1} \left[\overset{k-1}{\underset{m=2}{\pi}} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right)\right] f\left(y - \frac{c}{\beta}\right) f\left(y + \frac{\theta_1 - \theta_k}{\beta}\right) dy \\
&\leq 0.
\end{aligned}$$

Based on the preceding discussion, it follows that $\frac{\partial}{\partial \theta_{(1)}} P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\} \leq 0$.

Therefore, $P_{\underset{\sim}{\theta}}\{X_{[k]} - X_{[k-1]} > c\}$ is nonincreasing in $\theta_{(1)}$. Part b) is a result of repeated application of part a). □

2.1. <u>The Scale Parameter β Known Case</u>

Let $H(t)$ be the distribution function of $\frac{(X_1-\theta_1)-(X_2-\theta_2)}{\beta}$. Note that the distribution of $\frac{X_i-\theta_i}{\beta}$ is independent of θ_i and β. Thus the distribution $H(t)$ is independent of θ_1, θ_2 and β. For any real value t,

$$H(t) = \begin{cases} \int_0^{\infty} F(y+t)dF(y) & \text{if } t \geq 0, \\ \\ \int_{-t}^{\infty} F(y+t)dF(y) & \text{if } t < 0. \end{cases}$$

Also, $H(-t) = 1 - H(t)$ for all t. For each fixed $\alpha, 0 < \alpha < 1$, let $t_{\frac{\alpha}{2}}$ be the upper $\frac{\alpha}{2}$-quantile of the distribution $H(t)$. By the symmetric property of $H(t), t_{\frac{\alpha}{2}} > 0$. For this fixed α, define a nonnegative function $L_\alpha(t)$ on $[0, \infty)$ implicitly by

$$H(L_\alpha(t) - t) + H(-L_\alpha(t) - t) = \alpha \text{ for } t \geq t_{\frac{\alpha}{2}} \tag{2.3}$$

and $L_\alpha(t) = 0$ if $0 \leq t < t_{\frac{\alpha}{2}}$. One needs to prove that the function $L_\alpha(t)$ is well defined.

<u>Lemma 2.2</u>. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, the function $L_\alpha(t)$ defined implicitly by (2.3) always exists.

Proof: Let $h(t)$ be the density function of $H(t)$. Then $h(t)$ is symmetric about the point 0. Under the assumption, one can see that $h(t)$ is unimodal and $h(t_1) > h(t_2)$ if $|t_1| < |t_2|$.

For each fixed $t \geq t_{\frac{\alpha}{2}}$, define the function $M(c)$ for $c \geq 0$ as follows: $M(c) = H(c - t) + H(-c-t)$. Then, $M'(c) = h(c-t) - h(-c-t) > 0$ for $c > 0$ since $|c-t| < c+t$. Thus, $M(c)$ is strictly increasing in c. Now, $M(0) = 2H(-t) \leq 2H(-t_{\frac{\alpha}{2}}) = \alpha$ since $t \geq t_{\frac{\alpha}{2}}$. Also, $\lim_{c\to\infty} M(c) = 1$. By the continuity and strictly increasing property of $H(t)$, there exists a

unique $c > 0$ such that $M(c) = H(c-t)+H(-c-t) = \alpha$. We then denote that c by $L_\alpha(t)$. Thus, $L_\alpha(t)$ is well-defined. $\square$

Lemma 2.3. For given $0 < \alpha < 1$, the function $L_\alpha(t)$ is strictly increasing in t for $t \geq t_{\frac{\alpha}{2}}$.

Proof of the above lemma is straightforward.

Remark 2.1. a) For each fixed $\alpha, 0 < \alpha < 1$, by the definition of $L_\alpha(t)$, as $t \to \infty, L_\alpha(t) - t \to t_{1-\alpha}$ where $t_{1-\alpha}$ is the point such that $H(t_{1-\alpha}) = \alpha$. Since $t_{1-\alpha}$ is a fixed number. $L_\alpha(t) \to \infty$ as $t \to \infty$. Also, as $0 < \alpha \leq \frac{1}{2}, L_\alpha(t) - t < 0$ for all $t \geq t_{\frac{\alpha}{2}}$. This can be verified by noting that if $L_\alpha(t) \geq t$ for some $t \geq t_{\frac{\alpha}{2}}$, then, $\alpha = H(L_\alpha(t) - t) + H(-L_\alpha(t) - t) \geq H(0) + H(-L_\alpha(t) - t) > \frac{1}{2}$, which is a contradiction.

b) Since $L_\alpha(t)$ is strictly increasing in t for $t \geq t_{\frac{\alpha}{2}}$ and $L_\alpha(t) = 0$ for $0 \leq t < t_{\frac{\alpha}{2}}$. we may have a generalized inverse function of $L_\alpha(t)$ by letting $L_\alpha^{-1}(0) = t_{\frac{\alpha}{2}}$ and for $s > 0, L_\alpha^{-1}(s) = t$ if $L_\alpha(t) = s$. Note that $L_\alpha^{-1}(s)$ is strictly increasing in s.

In the following, we give a conservative $100(1-\alpha)\%$ lower confidence bound for $\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}$.

Theorem 2.1. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, $P_{\underset{\sim}{\theta}}\left\{\frac{\theta_{(k)}-\theta_{(k-1)}}{\beta} > L_\alpha\left(\frac{X_{[k]}-X_{[k-1]}}{\beta}\right)\right\} \geq 1-\alpha$ for all $\underset{\sim}{\theta}$.

Proof: By Lemma 2.1 and the definition of $L_\alpha(t)$, it follows that

$$\begin{aligned}
&P_{\underset{\sim}{\theta}}\left\{\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \leq L_\alpha\left(\frac{X_{[k]} - X_{[k-1]}}{\beta}\right)\right\} \\
&= P_{\underset{\sim}{\theta}}\left\{L_\alpha^{-1}\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right) \leq \frac{X_{[k]} - X_{[k-1]}}{\beta}\right\} \\
&\leq P_{\underset{\sim}{\theta}^0}\left\{L_\alpha^{-1}\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right) \leq \left|\frac{X_{(k)} - X_{(k-1)}}{\beta}\right|\right\} \\
&= H(L_\alpha(t_0) - t_0) + H(-L_\alpha(t_0) - t_0) \\
&= \alpha, \text{ by the definition of } L_\alpha(t),
\end{aligned}$$

where $t_0 = L_\alpha^{-1}(\frac{\theta_{(k)}-\theta_{(k-1)}}{\beta})$ and where $\underset{\sim}{\theta}^0$ and $X_{(i)}$ are defined previously. Thus,

$$P_{\underset{\sim}{\theta}}\left\{\frac{\theta_{(k)}-\theta_{(k-1)}}{\beta} > L_\alpha\left(\frac{X_{[k]}-X_{[k-1]}}{\beta}\right)\right\} \geq 1-\alpha \text{ for all } \underset{\sim}{\theta}. \qquad \square$$

The following theorem is a direct consequency of Theorem 2.1.

Theorem 2.2. Let $\hat{P}_L = \int_0^\infty \left[F\left(y + L_\alpha\left(\frac{X_{[k]}-X_{[k-1]}}{\beta}\right)\right)\right]^{k-1} dF(y)$. Then, under the assumption that $\log f(y)$ is concave on $(0,\infty)$,

$$P_{\underset{\sim}{\theta}}\{\text{PCS } (\underset{\sim}{\theta}) \geq \hat{P}_L\} \geq 1-\alpha \text{ for all } \underset{\sim}{\theta}.$$

That is, $\hat{P}_L$ is an at least $100(1-\alpha)\%$ lower confidence bound for the PCS $(\underset{\sim}{\theta})$.

2.2 The Scale Parameter β Unknown Case

When the scale parameter β is unknown, for each $i = 1,\ldots,k$, let $T_i = T(X_{i1},\ldots,X_{in})$ be a nonnegative function of $X_{i1},\ldots,X_{in}$, which depends on $X_{i1},\ldots,X_{in}$ only through the difference $X_{ij} - X_i, j = 1,\ldots,n$. That is, T is a location-invariant function. It is assumed that the function T is such that $T(cx_1,\ldots,cx_n) = cT(x_1,\ldots,x_n)$ for any positive value c. Also, let $S = S(T_1,\ldots,T_k)$ be a nonnegative function of $T_1,\ldots,T_k$ such that $S(ct_1,\ldots,ct_k) = cS(t_1,\ldots,t_k)$ for all $c > 0$. If for each $i = 1,\ldots,k, X_i$ is a complete sufficient statistic for the parameter θ_i, then T_i is independent of X_i since the distribution of T_i is independent of the parameter θ_i. Therefore, the distribution of S is independent of the parameters $\theta_1,\ldots,\theta_k$, and S is independent of $(X_1,\ldots,X_k)$. Also, by the preceding assumption, the distribution of $W = \frac{S}{\beta}$ is independent of the parameter β. Let $Q(w)$ denote the distribution of W.

For each fixed $0 < \alpha < 1$, let $t^*_{\frac{\alpha}{2}}$ be the point such that $\int_0^\infty H\left(-t^*_{\frac{\alpha}{2}}y\right)dQ(y) = \frac{\alpha}{2}$. Note that $t^*_{\frac{\alpha}{2}} > 0$. Define a nonnegative function $L^*_\alpha(t)$ on $[0,\infty)$ implicitly by

$$\int_0^\infty [H(L^*_\alpha(t) - ty) + H(-L^*_\alpha(t) - ty)]dQ(y) = \alpha \text{ for } t \geq t^*_{\frac{\alpha}{2}} \qquad (2.4)$$

and $L^*_\alpha(t) = 0$ if $0 \leq t < t^*_{\frac{\alpha}{2}}$. Analogous to Lemmas 2.2 and 2.3, we have the following lemma.

Lemma 2.4. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, $L^*_\alpha(t)$ always exists. Also. $L^*_\alpha(t)$ is strictly increasing in t for $t > t^*_{\frac{\alpha}{2}}$ and $L^*_\alpha\left(t^*_{\frac{\alpha}{2}}\right) = 0$.

Analogous to Remark 2.1.b, we let $L^{*^{-1}}_\alpha(\cdot)$ be the generalized inverse function of $L^*_\alpha(\cdot)$. It should be noted that $L^{*^{-1}}_\alpha(s)$ is strictly increasing in s for $s \geq 0$. Now, a conservative $100(1-\alpha)\%$ lower confidence bound for $(\theta_{(k)} - \theta_{(k-1)})/\beta$ is given as follows.

Theorem 2.3. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then,

$$P_{\underset{\sim}{\theta},\beta}\left\{\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} > L^*_\alpha\left(\frac{X_{[k]} - X_{[k-1]}}{S}\right)\right\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta} \text{ and } \beta.$$

Proof: By Lemma 2.1, it follows that

$$\begin{aligned}
&P_{\underset{\sim}{\theta},\beta}\left\{\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \leq L^*_\alpha\left(\frac{X_{[k]} - X_{[k-1]}}{S}\right)\right\} \\
&= P_{\underset{\sim}{\theta},\beta}\left\{L^{*^{-1}}_\alpha\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right) \leq \frac{X_{[k]} - X_{[k-1]}}{S}\right\} \\
&= P_{\underset{\sim}{\theta},\beta}\left\{L^{*^{-1}}_\alpha\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right)\frac{S}{\beta} \leq \frac{X_{[k]} - X_{[k-1]}}{\beta}\right\} \\
&\leq P_{\underset{\sim}{\theta}^0,\beta}\left\{L^{*^{-1}}_\alpha\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right)\frac{S}{\beta} \leq \left|\frac{X_{(k)} - X_{(k-1)}}{\beta}\right|\right\} \\
&= \int_0^\infty [H(L^*_\alpha(t_0) - t_0 y) + H(-L^*_\alpha(t_0) - t_0 y)]dQ(y) \\
&= \alpha
\end{aligned}$$

where $t_0 = L^{*^{-1}}_\alpha\left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}\right)$ and the last equality is obtained due to the definition of $L^*_\alpha(t)$. Thus, the proof of this theorem is complete. □

Theorem 2.4. Let $\hat{P}^*_L = \int_0^\infty \left[F\left(y + L^*_\alpha\left(\frac{X_{[k]} - X_{[k-1]}}{S}\right)\right)\right]^{k-1} dF(y)$. Assume that $\log f(y)$ is concave in $(0, \infty)$. Then

$$P_{\underset{\sim}{\theta},\beta}\left\{\text{PCS }(\underset{\sim}{\theta}) \geq \hat{P}^*_L\right\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta} \text{ and } \beta.$$

3. Selecting the Best Exponential Population

Let $X_{ij}, j = 1, \ldots, n$, be a sample of size n from a two-parameter exponential distribution with density function $g(x|\theta_i, \beta) = \beta^{-1} e^{-(x-\theta_i)/\beta} I_{(\theta_i, \infty)}(x), i = 1, \ldots, k$, where the common scale parameter β may be either known or unknown. The best population is the one associated with the largest truncated location parameter $\theta_{(k)}$. For each $i = 1, \ldots, k$, let $X_i = \min(X_{i1}, \ldots, X_{in})$. Based on $X_1, \ldots, X_k$, the natural selection rule selects the population yielding the largest sampled value $X_{[k]}$ as the best population. The corresponding PCS is:

$$
\begin{aligned}
\text{PCS}\,(\underset{\sim}{\theta}) &= \int_{y=0}^{\infty} \prod_{i=1}^{k-1} \left[1 - e^{-\left(y + \frac{n(\theta_{(k)} - \theta_{(i)})}{\beta} \right)} \right] e^{-y} dy \\
&\geq \int_{y=0}^{\infty} \left[1 - e^{-\left(y + \frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta} \right)} \right]^{k-1} e^{-y} dy.
\end{aligned} \tag{3.1}
$$

In order to find out a lower confidence bound for the PCS, we need to obtain a lower confidence bound for $\frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta}$. We consider two situations according to whether the common scale parameter β is known or unknown.

3.1 Lower Confidence Bound for PCS: β Known Case

Let $\mu_i = \frac{n\theta_i}{\beta}$ and $Y_i = \frac{nX_i}{\beta}$. Then $Y_i - \mu_i$ has an exponential distribution with density $f(y) = e^{-y} I_{(0,\infty)}(y)$. Let $H(t)$ be the distribution of $(Y_1 - \mu_1) - (Y_2 - \mu_2)$. Then,

$$
H(t) = \begin{cases} 1 - \frac{1}{2} e^{-t} & \text{if } t \geq 0, \\ \frac{1}{2} e^{t} & \text{if } t < 0. \end{cases}
$$

For fixed $\alpha \in (0.1)$, let $t_{\frac{\alpha}{2}}$ denote the upper $\frac{\alpha}{2}$-quantile of $H(t)$. Then, $t_{\frac{\alpha}{2}} = -\ell n \alpha$. Define function $L_\alpha(t)$ on $(0, \infty)$ such that $H(L_\alpha(t) - t) + H(-L_\alpha(t) - t) = \alpha$ for $t \geq t_{\frac{\alpha}{2}}$, and $L_\alpha(t) = 0$ if $0 \leq t < t_{\frac{\alpha}{2}}$. Since higher confidence statement is always desirable, we

need to consider only $\alpha \in (0, \frac{1}{2})$. By Remark 2.1.b, $L_\alpha(t) - t < 0$ for all $t > 0$. Thus, straightforward computation gives that $L_\alpha(t) = \ell n\left[\alpha e^t + \sqrt{\alpha^2 e^{2t} - 1}\right]$. From Theorem 2.1, letting $\tilde{\Delta} = \frac{n(X_{[k]} - X_{[k-1]})}{\beta}$, we have

$$P_{\underset{\sim}{\theta}}\left\{\frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta} > L_\alpha(\tilde{\Delta})\right\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta}.$$

Letting $\hat{P}_L = \int_{y=0}^{\infty}[1 - \exp(-y - L_\alpha(\tilde{\Delta}))]^{k-1} e^{-y} dy$, we then have:

$$P_{\underset{\sim}{\theta}}\{\text{PCS } (\underset{\sim}{\theta}) \geq \hat{P}_L\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta}.$$

3.2 <u>Lower Confidence Bound for PCS: β Unknown Case</u>

When the common scale parameter β is unknown, let $S = \sum_{i=1}^{k} \sum_{j=1}^{n} \frac{(X_{ij} - X_i)}{k(n-1)}$. Then S is independent of $X_1, \ldots, X_k$ and $\frac{k(n-1)S}{\beta}$ has a gamma distribution with shape parameter $m = k(n-1)$ and scale parameter 1. Let $Q_m(y)$ denote the distribution of $\frac{S}{\beta}$. For $0 < \alpha < 1$, let $t^*_{\frac{\alpha}{2}}$ be the point such that $\int_0^\infty H\left(-t^*_{\frac{\alpha}{2}} y\right) dQ_m(y) = \frac{\alpha}{2}$. Straightforward computation yields $t^*_{\frac{\alpha}{2}} = m\left(\alpha^{-\frac{1}{m}} - 1\right)$. The function $L^*_\alpha(t)$ is then implicitly defined by

$$\int_0^\infty [H(L^*_\alpha(t) - yt) + H(-L^*_\alpha(t) - yt)] dQ_m(y) = \alpha \text{ for } t \geq t^*_{\frac{\alpha}{2}}$$

and $L^*_\alpha(t) = 0$ for $0 \leq t < t^*_{\frac{\alpha}{2}}$. Thus, for $t > t^*_{\frac{\alpha}{2}}, L^*_\alpha(t)$ is such that

$$\begin{aligned} &\int_0^{L^*_\alpha(t)/t} \left[1 - \frac{1}{2} e^{-L^*_\alpha(t) + yt}\right] dQ_m(y) \\ &+ \int_{L^*(t)/t}^{\infty} \frac{1}{2} e^{L^*_\alpha(t) - yt} dQ_m(y) + \int_0^\infty \frac{1}{2} e^{-L^*_\alpha(t) - yt} dQ_m(y) = \alpha. \end{aligned}$$

Also, from Theorem 2.3,

$$P_{\underset{\sim}{\theta}, \beta}\left\{\frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta} > L^*_\alpha\left(\frac{n(X_{[k]} - X_{[k-1]})}{S}\right)\right\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta} \text{ and } \beta.$$

Letting $\Delta^* = \frac{n(X_{[k]} - X_{[k-1]})}{S}$ and $\hat{P}_L^* = \int_0^\infty [1 - \exp(-y - L_\alpha^*(\Delta^*)]^{k-1} e^{-y} dy$, we then have

$$P_{\underset{\sim}{\theta},\beta}\{\text{PCS }(\underset{\sim}{\theta}) \geq \hat{P}_L^*\} \geq 1 - \alpha \text{ for all } \underset{\sim}{\theta} \text{ and } \beta.$$

The values of the function $L_\alpha^*(t)$ are given in Tables 1 and 2 for $\alpha = 0.05, 0.10$ and for selected values of m and $t \geq t_{\frac{\alpha}{2}}^*$. Note that when m is sufficiently large, $t_{\frac{\alpha}{2}}^* \approx -\ell n\alpha$ and $L_\alpha^*(t) \approx \ell n\left[\alpha e^t + \sqrt{\alpha^2 e^{2t} - 1}\right]$ for $t > t_{\frac{\alpha}{2}}^*$.

4. <u>An Illustrative Example</u>

We use the insulating fluid example (taken from Table 4.1, page 462 of Nelson (1982)) to illustrate the way to implement the proposed procedure. There are six groups of insulating fluid. The purpose is to identify which group of insulating fluid has the largest guaranteed life-time when subjected to high voltage stress. Ten items from each group are put in a life-test experiment which is subject to high voltage stress. It is assumed that the distribution of the life-time for each insulating fluid is a two-parameter exponential with common unknown scale parameter β. The times to breakdown in minutes is shown in the following.

Table 3. Times to Breakdown

Group	1	2	3	4	5	6
	1.89	1.30	1.99	1.17	8.11	2.12
	4.03	2.75	0.64	3.87	3.17	3.97
	1.54	0.00	2.15	2.80	5.55	1.56
	0.31	2.17	1.08	0.70	0.80	1.49
	0.66	0.66	2.57	3.82	0.20	8.71
	1.70	0.55	0.93	0.02	1.13	2.10
	2.17	0.18	4.75	0.50	6.63	7.21
	1.82	10.60	0.82	3.72	1.08	3.83
	9.99	1.63	2.06	0.06	2.44	1.34
	2.24	0.71	0.49	3.57	0.78	5.13
X_i	0.31	0.00	0.49	0.02	0.20	1.34

From Table 3, we obtain: $X_1 = 0.31, X_2 = 0.00, X_3 = 0.49, X_4 = 0.02, X_5 = 0.20$ and

$X_6 = 1.34$. According to the natural selection rule, we select Group 6 as the best group. Then, a reasonable question is: what kind of confidence statement can be made regarding the PCS? For this purpose, based on the above given data, we have $\frac{n(X_{[6]} - X_{[5]})}{S} = 3.8455$. For different α values, the $100(1-\alpha)\%$ lower confidence bounds $\hat{P}_L^*$ of the PCS are computed and given as follows:

α	0.05	0.10	0.15	0.20	0.25
$\hat{P}_L^*$	0.5356	0.7373	0.8166	0.8591	0.8856

Thus, we can claim, for example, that with at least 85 percent confidence, PCS $\geq \hat{P}_L^* =$ 0.8166.

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Table 1. Values of $L^*_\alpha(t)$ for $\alpha = 0.05$

		$t - t^*_{\frac{\alpha}{2}}(m)$													
m	$t^*_{\frac{\alpha}{2}}(m)$	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700
5	4.103	0.235	0.334	0.410	0.475	0.532	0.585	0.634	0.680	0.724	0.765	0.805	0.843	0.881	0.917
6	3.885	0.247	0.351	0.432	0.500	0.561	0.617	0.669	0.718	0.764	0.808	0.851	0.892	0.932	0.970
7	3.739	0.256	0.364	0.448	0.519	0.583	0.641	0.695	0.747	0.795	0.841	0.886	0.929	0.971	1.011
8	3.634	0.263	0.374	0.461	0.534	0.600	0.660	0.716	0.769	0.819	0.867	0.914	0.958	1.002	1.044
9	3.555	0.269	0.382	0.471	0.546	0.614	0.675	0.733	0.787	0.839	0.888	0.936	0.982	1.027	1.071
10	3.493	0.274	0.389	0.479	0.556	0.625	0.688	0.747	0.802	0.855	0.906	0.954	1.002	1.048	1.092
11	3.443	0.277	0.395	0.486	0.564	0.634	0.698	0.758	0.815	0.868	0.920	0.970	1.018	1.065	1.111
12	3.403	0.281	0.399	0.492	0.571	0.642	0.707	0.768	0.825	0.880	0.932	0.983	1.032	1.080	1.127
13	3.369	0.283	0.403	0.497	0.577	0.649	0.714	0.776	0.834	0.890	0.943	0.995	1.044	1.093	1.140
14	3.340	0.286	0.407	0.501	0.582	0.654	0.721	0.783	0.842	0.898	0.952	1.004	1.055	1.104	1.152
15	3.316	0.288	0.410	0.505	0.586	0.660	0.727	0.790	0.849	0.906	0.960	1.013	1.064	1.114	1.162
16	3.295	0.290	0.412	0.508	0.590	0.664	0.732	0.795	0.855	0.913	0.968	1.021	1.072	1.123	1.172
17	3.276	0.291	0.415	0.511	0.594	0.668	0.736	0.800	0.861	0.919	0.974	1.028	1.080	1.130	1.180
18	3.259	0.293	0.417	0.514	0.597	0.672	0.740	0.805	0.866	0.924	0.980	1.034	1.086	1.137	1.187
19	3.245	0.294	0.419	0.516	0.600	0.675	0.744	0.809	0.870	0.929	0.985	1.039	1.092	1.144	1.194
20	3.232	0.295	0.420	0.518	0.602	0.678	0.747	0.812	0.874	0.933	0.990	1.044	1.098	1.149	1.200
21	3.220	0.296	0.422	0.520	0.605	0.681	0.750	0.816	0.878	0.937	0.994	1.049	1.102	1.155	1.206
22	3.209	0.297	0.423	0.522	0.607	0.683	0.753	0.819	0.881	0.941	0.998	1.053	1.107	1.159	1.211
23	3.200	0.298	0.425	0.524	0.609	0.685	0.756	0.822	0.884	0.944	1.001	1.057	1.111	1.164	1.215
24	3.191	0.299	0.426	0.525	0.611	0.687	0.758	0.824	0.887	0.947	1.005	1.061	1.115	1.168	1.220
25	3.183	0.300	0.427	0.527	0.612	0.689	0.760	0.827	0.889	0.950	1.008	1.064	1.118	1.171	1.223
30	3.150	0.303	0.432	0.532	0.619	0.697	0.769	0.836	0.900	0.961	1.020	1.077	1.132	1.187	1.240
40	3.111	0.307	0.437	0.539	0.627	0.707	0.780	0.848	0.913	0.976	1.036	1.094	1.151	1.206	1.260
50	3.087	0.309	0.441	0.544	0.633	0.713	0.786	0.856	0.921	0.985	1.045	1.104	1.162	1.218	1.273
60	3.072	0.311	0.443	0.547	0.636	0.717	0.791	0.861	0.927	0.990	1.052	1.111	1.169	1.226	1.281
120	3.033	0.315	0.449	0.554	0.645	0.727	0.802	0.873	0.941	1.006	1.068	1.129	1.188	1.246	1.303
∞	2.996	0.319	0.455	0.562	0.654	0.737	0.814	0.886	0.955	1.021	1.085	1.147	1.208	1.267	1.325

Table 1. Values of $L_{\alpha}^{*}(t)$ for $\alpha = 0.05$ (cont.)

		$t - t_{\frac{\alpha}{2}}^{*}(m)$													
m	$t_{\frac{\alpha}{2}}^{*}(m)$	0.750	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400
5	4.103	0.952	0.986	1.020	1.053	1.085	1.117	1.148	1.178	1.208	1.238	1.268	1.297	1.325	1.354
6	3.885	1.008	1.045	1.081	1.116	1.150	1.184	1.218	1.251	1.283	1.316	1.347	1.379	1.410	1.440
7	3.739	1.051	1.090	1.128	1.165	1.201	1.237	1.273	1.307	1.342	1.376	1.409	1.443	1.475	1.508
8	3.634	1.085	1.126	1.165	1.204	1.242	1.279	1.316	1.353	1.389	1.424	1.459	1.494	1.529	1.563
9	3.555	1.113	1.155	1.196	1.236	1.275	1.314	1.352	1.390	1.427	1.464	1.500	1.537	1.572	1.608
10	3.493	1.136	1.179	1.221	1.262	1.303	1.343	1.382	1.421	1.459	1.497	1.535	1.572	1.609	1.646
11	3.443	1.156	1.199	1.242	1.284	1.326	1.367	1.407	1.447	1.487	1.526	1.564	1.602	1.640	1.678
12	3.403	1.172	1.217	1.261	1.304	1.346	1.388	1.429	1.470	1.510	1.550	1.589	1.628	1.667	1.706
13	3.369	1.186	1.232	1.276	1.320	1.363	1.406	1.448	1.489	1.530	1.571	1.611	1.651	1.690	1.730
14	3.340	1.199	1.245	1.290	1.335	1.378	1.422	1.464	1.506	1.548	1.589	1.630	1.671	1.711	1.751
15	3.316	1.210	1.257	1.302	1.347	1.392	1.435	1.479	1.522	1.564	1.606	1.647	1.688	1.729	1.770
16	3.295	1.220	1.267	1.313	1.359	1.404	1.448	1.492	1.535	1.578	1.620	1.662	1.704	1.745	1.786
17	3.276	1.228	1.276	1.323	1.369	1.414	1.459	1.503	1.547	1.590	1.633	1.676	1.718	1.760	1.802
18	3.259	1.236	1.284	1.331	1.378	1.424	1.469	1.514	1.558	1.602	1.645	1.688	1.731	1.773	1.815
19	3.245	1.243	1.292	1.339	1.386	1.432	1.478	1.523	1.568	1.612	1.656	1.699	1.742	1.785	1.827
20	3.232	1.250	1.298	1.346	1.394	1.440	1.486	1.532	1.577	1.621	1.665	1.709	1.753	1.796	1.839
21	3.220	1.256	1.305	1.353	1.400	1.447	1.494	1.539	1.585	1.630	1.674	1.718	1.762	1.806	1.849
22	3.209	1.261	1.310	1.359	1.407	1.454	1.500	1.547	1.592	1.637	1.682	1.727	1.771	1.815	1.858
23	3.200	1.266	1.315	1.364	1.412	1.460	1.507	1.553	1.599	1.645	1.690	1.734	1.779	1.823	1.867
24	3.191	1.270	1.320	1.369	1.418	1.465	1.513	1.559	1.605	1.651	1.697	1.742	1.786	1.831	1.875
25	3.183	1.274	1.325	1.374	1.423	1.471	1.518	1.565	1.611	1.657	1.703	1.748	1.793	1.838	1.882
30	3.150	1.291	1.343	1.393	1.442	1.491	1.540	1.588	1.635	1.682	1.729	1.775	1.821	1.867	1.912
40	3.111	1.313	1.365	1.417	1.468	1.518	1.568	1.617	1.666	1.714	1.762	1.810	1.857	1.905	1.951
50	3.087	1.327	1.380	1.432	1.484	1.535	1.585	1.635	1.685	1.734	1.783	1.831	1.880	1.928	1.976
60	3.072	1.336	1.389	1.442	1.494	1.546	1.597	1.647	1.698	1.747	1.797	1.846	1.895	1.944	1.992
120	3.033	1.358	1.413	1.468	1.521	1.574	1.627	1.679	1.730	1.782	1.833	1.883	1.934	1.984	2.034
∞	2.996	1.382	1.438	1.494	1.549	1.603	1.657	1.711	1.764	1.817	1.870	1.922	1.974	2.026	2.078

Table 1. Values of $L^*_\alpha(t)$ for $\alpha = 0.05$ (cont.)

		$t - t^*_{\frac{\alpha}{2}}(m)$											
m	$t^*_{\frac{\alpha}{2}}(m)$	1.450	1.500	1.550	1.600	1.650	1.700	1.750	1.800	1.850	1.900	1.950	2.000
5	4.103	1.382	1.410	1.437	1.465	1.492	1.519	1.545	1.572	1.598	1.624	1.650	1.676
6	3.885	1.471	1.501	1.531	1.560	1.590	1.619	1.648	1.677	1.705	1.734	1.762	1.790
7	3.739	1.540	1.572	1.604	1.635	1.667	1.698	1.729	1.759	1.790	1.820	1.850	1.880
8	3.634	1.596	1.630	1.663	1.696	1.729	1.762	1.794	1.826	1.858	1.890	1.922	1.953
9	3.555	1.643	1.678	1.712	1.747	1.781	1.815	1.848	1.882	1.915	1.948	1.981	2.014
10	3.493	1.682	1.718	1.754	1.789	1.824	1.859	1.894	1.929	1.963	1.998	2.032	2.066
11	3.443	1.715	1.752	1.789	1.825	1.862	1.898	1.933	1.969	2.005	2.040	2.075	2.110
12	3.403	1.744	1.782	1.819	1.857	1.894	1.931	1.967	2.004	2.040	2.076	2.113	2.148
13	3.369	1.769	1.807	1.846	1.884	1.922	1.959	1.997	2.034	2.072	2.109	2.145	2.182
14	3.340	1.791	1.830	1.869	1.908	1.946	1.985	2.023	2.061	2.099	2.137	2.174	2.212
15	3.316	1.810	1.850	1.890	1.929	1.968	2.007	2.046	2.085	2.124	2.162	2.200	2.238
16	3.295	1.827	1.868	1.908	1.948	1.988	2.028	2.067	2.106	2.145	2.184	2.223	2.262
17	3.276	1.843	1.884	1.925	1.965	2.006	2.046	2.086	2.126	2.165	2.205	2.244	2.283
18	3.259	1.857	1.898	1.940	1.981	2.022	2.062	2.103	2.143	2.183	2.223	2.263	2.302
19	3.245	1.870	1.912	1.953	1.995	2.036	2.077	2.118	2.159	2.199	2.240	2.280	2.320
20	3.232	1.881	1.923	1.966	2.007	2.049	2.091	2.132	2.173	2.214	2.255	2.296	2.336
21	3.220	1.892	1.934	1.977	2.019	2.061	2.103	2.145	2.186	2.228	2.269	2.310	2.351
22	3.209	1.901	1.945	1.987	2.030	2.072	2.115	2.157	2.198	2.240	2.282	2.323	2.364
23	3.200	1.910	1.954	1.997	2.040	2.083	2.125	2.167	2.210	2.252	2.293	2.335	2.377
24	3.191	1.919	1.962	2.006	2.049	2.092	2.135	2.177	2.220	2.262	2.304	2.346	2.388
25	3.183	1.926	1.970	2.014	2.057	2.101	2.144	2.187	2.229	2.272	2.314	2.357	2.399
30	3.150	1.958	2.003	2.048	2.092	2.137	2.181	2.225	2.269	2.313	2.356	2.400	2.443
40	3.111	1.998	2.045	2.091	2.137	2.183	2.229	2.274	2.320	2.365	2.410	2.455	2.500
50	3.087	2.023	2.071	2.118	2.165	2.212	2.258	2.305	2.351	2.398	2.444	2.490	2.536
60	3.072	2.040	2.088	2.136	2.184	2.231	2.279	2.326	2.373	2.420	2.467	2.514	2.560
120	3.033	2.084	2.133	2.183	2.232	2.281	2.330	2.379	2.428	2.477	2.526	2.574	2.623
∞	2.996	2.129	2.180	2.232	2.283	2.334	2.385	2.436	2.486	2.537	2.588	2.638	2.689

Table 2. Values of $L^*_\alpha(t)$ for $\alpha = 0.10$

		$t - t^*_{\frac{\alpha}{2}}(m)$													
m	$t^*_{\frac{\alpha}{2}}(m)$	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700
5	2.924	0.252	0.358	0.440	0.510	0.572	0.629	0.682	0.732	0.780	0.825	0.869	0.911	0.952	0.992
6	2.807	0.262	0.372	0.458	0.531	0.596	0.656	0.711	0.764	0.813	0.861	0.907	0.951	0.994	1.036
7	2.726	0.270	0.383	0.471	0.547	0.614	0.676	0.733	0.787	0.839	0.888	0.936	0.982	1.027	1.070
8	2.668	0.275	0.391	0.482	0.559	0.628	0.691	0.750	0.806	0.859	0.910	0.959	1.006	1.052	1.097
9	2.624	0.280	0.398	0.490	0.568	0.639	0.703	0.764	0.821	0.875	0.927	0.977	1.026	1.073	1.119
10	2.589	0.283	0.403	0.496	0.576	0.648	0.714	0.775	0.833	0.888	0.941	0.992	1.042	1.090	1.137
11	2.561	0.286	0.407	0.502	0.583	0.655	0.722	0.784	0.843	0.899	0.953	1.005	1.055	1.104	1.152
12	2.538	0.289	0.411	0.507	0.588	0.662	0.729	0.792	0.852	0.908	0.963	1.016	1.067	1.116	1.165
13	2.519	0.291	0.414	0.511	0.593	0.667	0.735	0.799	0.859	0.916	0.972	1.025	1.076	1.127	1.176
14	2.503	0.293	0.417	0.514	0.597	0.672	0.740	0.805	0.865	0.923	0.979	1.033	1.085	1.136	1.186
15	2.489	0.295	0.419	0.517	0.601	0.676	0.745	0.810	0.871	0.929	0.986	1.040	1.093	1.144	1.194
16	2.477	0.296	0.422	0.520	0.604	0.680	0.749	0.814	0.876	0.935	0.991	1.046	1.099	1.151	1.202
17	2.466	0.297	0.423	0.522	0.607	0.683	0.753	0.818	0.880	0.940	0.997	1.052	1.105	1.157	1.208
18	2.456	0.299	0.425	0.524	0.609	0.686	0.756	0.822	0.884	0.944	1.001	1.057	1.111	1.163	1.214
19	2.448	0.300	0.427	0.526	0.611	0.688	0.759	0.825	0.888	0.948	1.005	1.061	1.115	1.168	1.220
20	2.440	0.301	0.428	0.528	0.614	0.691	0.761	0.828	0.891	0.951	1.009	1.065	1.120	1.173	1.225
21	2.434	0.301	0.429	0.529	0.615	0.693	0.764	0.831	0.894	0.954	1.013	1.069	1.124	1.177	1.229
22	2.427	0.302	0.430	0.531	0.617	0.695	0.766	0.833	0.897	0.957	1.016	1.072	1.127	1.181	1.233
23	2.422	0.303	0.431	0.532	0.619	0.696	0.768	0.835	0.899	0.960	1.019	1.075	1.131	1.184	1.237
24	2.417	0.304	0.432	0.533	0.620	0.698	0.770	0.837	0.901	0.962	1.021	1.078	1.134	1.188	1.240
25	2.412	0.304	0.433	0.534	0.621	0.699	0.772	0.839	0.903	0.965	1.024	1.081	1.136	1.191	1.244
30	2.393	0.307	0.437	0.539	0.627	0.706	0.778	0.847	0.912	0.974	1.033	1.091	1.148	1.203	1.257
40	2.370	0.310	0.441	0.544	0.633	0.713	0.787	0.856	0.922	0.985	1.046	1.105	1.162	1.218	1.273
50	2.356	0.311	0.444	0.548	0.637	0.718	0.792	0.862	0.929	0.992	1.054	1.113	1.171	1.228	1.283
60	2.347	0.313	0.446	0.550	0.640	0.721	0.796	0.866	0.933	0.997	1.059	1.119	1.177	1.234	1.290
120	2.325	0.316	0.450	0.556	0.647	0.729	0.805	0.876	0.944	1.009	1.072	1.133	1.192	1.250	1.307
∞	2.303	0.319	0.455	0.562	0.654	0.737	0.814	0.886	0.955	1.021	1.085	1.147	1.208	1.267	1.325

Table 2. Values of $L_{\alpha}^{*}(t)$ for $\alpha = 0.10$ (cont.)

		$t - t_{\frac{\alpha}{2}}^{*}(m)$													
m	$t_{\frac{\alpha}{2}}^{*}(m)$	0.750	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400
5	2.924	1.030	1.068	1.105	1.142	1.178	1.213	1.248	1.282	1.316	1.349	1.382	1.415	1.447	1.479
6	2.807	1.077	1.117	1.156	1.195	1.232	1.270	1.306	1.343	1.378	1.414	1.449	1.484	1.518	1.552
7	2.726	1.113	1.154	1.195	1.235	1.275	1.313	1.352	1.390	1.427	1.464	1.501	1.537	1.573	1.608
8	2.668	1.141	1.184	1.226	1.267	1.308	1.348	1.388	1.427	1.466	1.504	1.542	1.579	1.617	1.654
9	2.624	1.164	1.208	1.251	1.294	1.335	1.377	1.417	1.458	1.497	1.537	1.576	1.614	1.653	1.691
10	2.589	1.183	1.228	1.272	1.315	1.358	1.400	1.442	1.483	1.524	1.564	1.604	1.644	1.683	1.722
11	2.561	1.199	1.244	1.289	1.334	1.377	1.420	1.463	1.505	1.546	1.588	1.628	1.669	1.709	1.749
12	2.538	1.212	1.259	1.304	1.349	1.394	1.438	1.481	1.523	1.566	1.608	1.649	1.690	1.731	1.771
13	2.519	1.224	1.271	1.317	1.363	1.408	1.452	1.496	1.540	1.582	1.625	1.667	1.709	1.750	1.791
14	2.503	1.234	1.282	1.329	1.375	1.420	1.465	1.510	1.554	1.597	1.640	1.683	1.725	1.767	1.809
15	2.489	1.243	1.291	1.339	1.385	1.431	1.477	1.522	1.566	1.610	1.654	1.697	1.740	1.782	1.824
16	2.477	1.251	1.300	1.348	1.395	1.441	1.487	1.532	1.577	1.622	1.666	1.709	1.752	1.795	1.838
17	2.466	1.258	1.307	1.355	1.403	1.450	1.496	1.542	1.587	1.632	1.676	1.720	1.764	1.807	1.851
18	2.456	1.265	1.314	1.363	1.410	1.458	1.504	1.550	1.596	1.641	1.686	1.730	1.774	1.818	1.862
19	2.448	1.270	1.320	1.369	1.417	1.465	1.512	1.558	1.604	1.650	1.695	1.739	1.784	1.828	1.872
20	2.440	1.276	1.326	1.375	1.423	1.471	1.518	1.565	1.611	1.657	1.703	1.748	1.792	1.837	1.881
21	2.434	1.280	1.331	1.380	1.429	1.477	1.524	1.571	1.618	1.664	1.710	1.755	1.800	1.845	1.890
22	2.427	1.285	1.335	1.385	1.434	1.482	1.530	1.577	1.624	1.671	1.717	1.762	1.808	1.853	1.897
23	2.422	1.289	1.339	1.389	1.439	1.487	1.535	1.583	1.630	1.676	1.723	1.769	1.814	1.859	1.904
24	2.417	1.292	1.343	1.393	1.443	1.492	1.540	1.588	1.635	1.682	1.728	1.775	1.820	1.866	1.911
25	2.412	1.296	1.347	1.397	1.447	1.496	1.544	1.592	1.640	1.687	1.734	1.780	1.826	1.872	1.917
30	2.393	1.309	1.361	1.413	1.463	1.513	1.562	1.611	1.659	1.707	1.755	1.802	1.849	1.896	1.942
40	2.370	1.327	1.380	1.432	1.484	1.535	1.585	1.635	1.685	1.734	1.782	1.831	1.879	1.927	1.974
50	2.356	1.338	1.391	1.444	1.496	1.548	1.599	1.650	1.700	1.750	1.799	1.848	1.897	1.946	1.994
60	2.347	1.345	1.399	1.452	1.505	1.557	1.609	1.660	1.710	1.761	1.811	1.860	1.910	1.959	2.008
120	2.325	1.363	1.418	1.473	1.527	1.580	1.633	1.685	1.737	1.788	1.840	1.891	1.941	1.992	2.042
∞	2.303	1.382	1.438	1.494	1.549	1.603	1.657	1.711	1.764	1.817	1.870	1.922	1.974	2.026	2.078

Table 2. Values of $L^*_\alpha(t)$ for $\alpha = 0.10$ (cont.)

		$t - t^*_{\frac{\alpha}{2}}(m)$											
m	$t^*_{\frac{\alpha}{2}}(m)$	1.450	1.500	1.550	1.600	1.650	1.700	1.750	1.800	1.850	1.900	1.950	2.000
5	2.924	1.511	1.543	1.574	1.605	1.636	1.666	1.697	1.727	1.757	1.787	1.817	1.846
6	2.807	1.586	1.619	1.652	1.685	1.718	1.751	1.783	1.815	1.847	1.879	1.911	1.943
7	2.726	1.644	1.679	1.714	1.748	1.783	1.817	1.851	1.884	1.918	1.952	1.985	2.018
8	2.668	1.690	1.727	1.763	1.799	1.834	1.870	1.905	1.940	1.975	2.010	2.045	2.079
9	2.624	1.729	1.766	1.803	1.840	1.877	1.914	1.950	1.986	2.022	2.058	2.094	2.130
10	2.589	1.761	1.799	1.837	1.875	1.913	1.951	1.988	2.025	2.062	2.099	2.136	2.172
11	2.561	1.788	1.827	1.866	1.905	1.944	1.982	2.020	2.058	2.096	2.134	2.171	2.209
12	2.538	1.812	1.852	1.891	1.931	1.970	2.009	2.048	2.087	2.126	2.164	2.202	2.240
13	2.519	1.832	1.873	1.913	1.953	1.993	2.033	2.073	2.112	2.151	2.190	2.229	2.268
14	2.503	1.850	1.891	1.932	1.973	2.014	2.054	2.094	2.134	2.174	2.214	2.253	2.293
15	2.489	1.866	1.908	1.949	1.991	2.032	2.073	2.113	2.154	2.194	2.234	2.274	2.314
16	2.477	1.881	1.923	1.965	2.006	2.048	2.089	2.130	2.171	2.212	2.253	2.293	2.334
17	2.466	1.893	1.936	1.978	2.021	2.062	2.104	2.146	2.187	2.229	2.270	2.311	2.351
18	2.456	1.905	1.948	1.991	2.033	2.076	2.118	2.160	2.202	2.243	2.285	2.326	2.367
19	2.448	1.915	1.959	2.002	2.045	2.088	2.130	2.172	2.215	2.257	2.299	2.340	2.382
20	2.440	1.925	1.969	2.012	2.055	2.098	2.141	2.184	2.227	2.269	2.311	2.353	2.395
21	2.434	1.934	1.978	2.022	2.065	2.108	2.152	2.195	2.237	2.280	2.323	2.365	2.407
22	2.427	1.942	1.986	2.030	2.074	2.118	2.161	2.204	2.247	2.290	2.333	2.376	2.419
23	2.422	1.949	1.994	2.038	2.082	2.126	2.170	2.213	2.257	2.300	2.343	2.386	2.429
24	2.417	1.956	2.001	2.045	2.090	2.134	2.178	2.222	2.265	2.309	2.352	2.395	2.439
25	2.412	1.962	2.007	2.052	2.097	2.141	2.185	2.229	2.273	2.317	2.361	2.404	2.448
30	2.393	1.988	2.034	2.080	2.125	2.171	2.216	2.261	2.306	2.350	2.395	2.439	2.484
40	2.370	2.022	2.069	2.116	2.163	2.209	2.256	2.302	2.348	2.394	2.440	2.486	2.532
50	2.356	2.042	2.090	2.138	2.186	2.233	2.280	2.328	2.375	2.422	2.468	2.515	2.562
60	2.347	2.056	2.105	2.153	2.201	2.249	2.297	2.345	2.393	2.440	2.487	2.535	2.582
120	2.325	2.092	2.142	2.192	2.241	2.291	2.340	2.389	2.438	2.488	2.536	2.585	2.634
∞	2.303	2.129	2.180	2.232	2.283	2.334	2.385	2.436	2.486	2.537	2.588	2.638	2.689

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We are concerned with deriving lower confidence bounds for the probability of a correct selection in truncated location-parameter models. Two cases are considered according to whether the scale parameter is known or unknown. For each case, a lower confidence bound for the difference between the best and the second best is obtained. These lower confidence bounds are used to construct lower confidence bounds for the probability of a correct selection. The results are then applied to the problem of selecting the best exponential population having the largest truncated location-parameter. Useful tables are provided for implementing the proposed methods.

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